Bisimulation partitioning and partition maintenance

on very large directed acyclic graphs

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Overview

Why bisimulation?

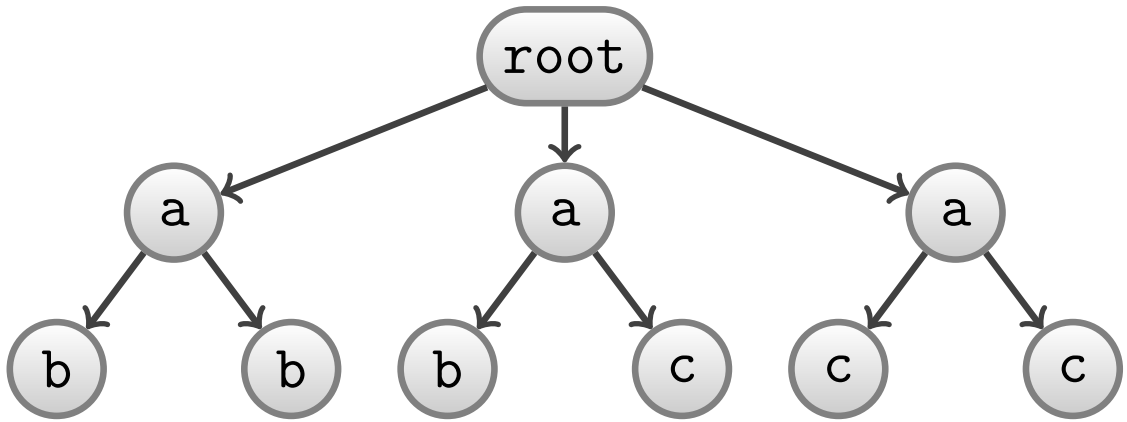
- For my Master: Introduced formally in 6 courses
- Played a role in 4 other Master courses

Usage

- Some data represented by graph
- Operation retrieve useful information from this data
- Operation is slow on large graphs
- Group together all data in the graph which are equivalent with respect to the operation

Example: Querying XML data

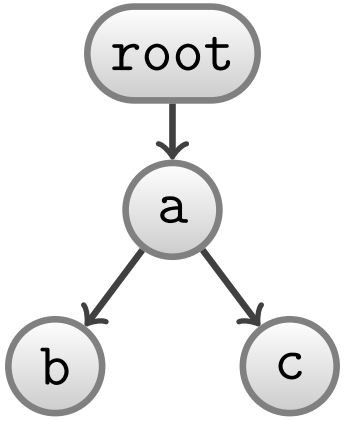
```
<root>
   <a>
      <b/> <b/>
   </a>
   <a>
      <b/> <c/>
   </a>
   <a>
      <c/> <c/>
   </a>
</root>
```



Query 1 is the path `root/a/b` reachable?

Query 2 give all nodes reachable by path `root/a/b`.

Grouping nodes using backward bisimulation



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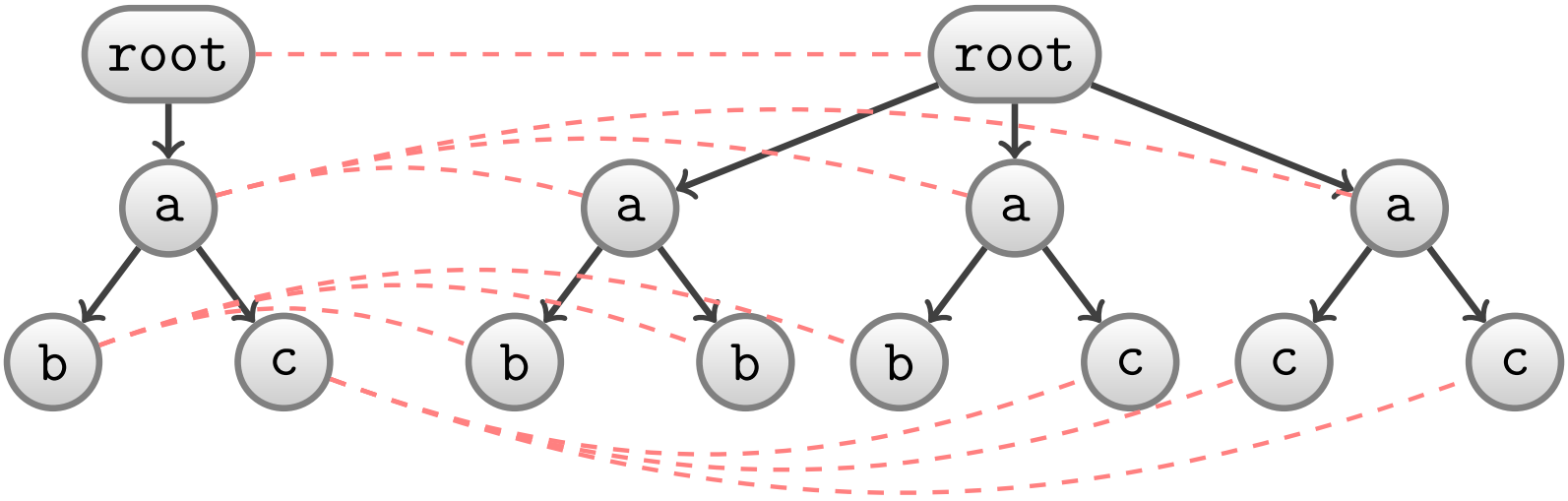
Graph index



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Problem statement

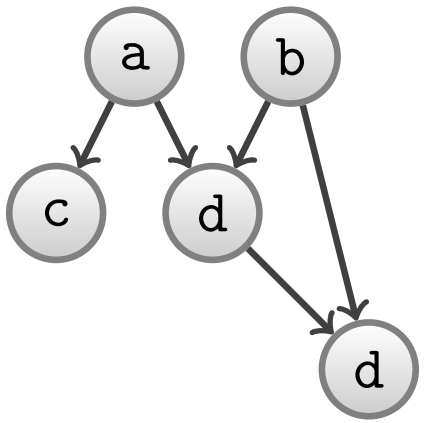
1. Bisimulation partitioning
2. Partition maintenance

Graph

Definition

A (node labeled) graph is represented by a triple $G =< N, E, l >$:

1. N is a set of nodes,
2. $E \subseteq N \times N$ is a directed edge relation,
3. l is a node-label function



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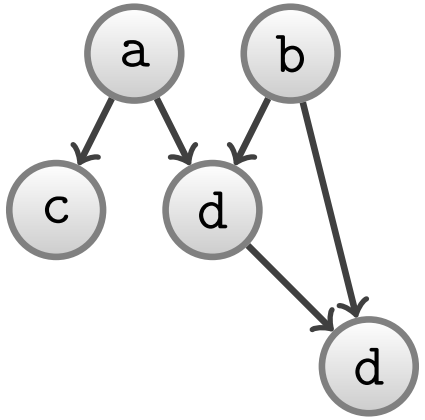


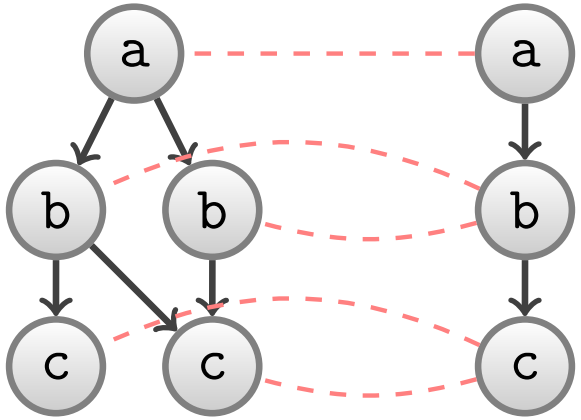
- We restrict ourselves to acyclic graphs

Bisimulation

Definition

Node n bisimulates node m; denoted as $n \approx m$; if and only if:

1. The nodes have the same label,
2. Every child n' of n is bisimulated by a child m' of m, and
3. Every child m' of m is bisimulated by a child n' of n.



Bisimulation partitioning

- Already very well studied
- Fast general algorithm by Paige and Tarjan [1987]
  - Runtime complexity: $O(|E| \log(|N|))$
  - Memory usage: $O(|N| + |E|)$

Internal vs External

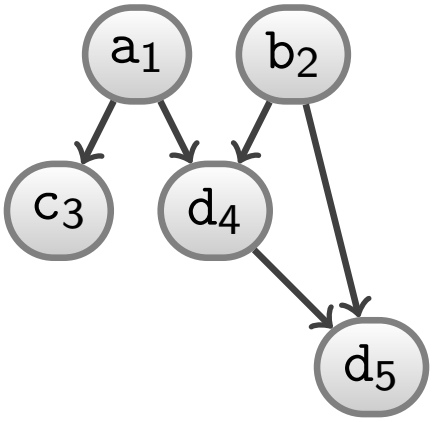
Cost for reading or writing data from hard disk drive:

Seek move to the correct position on the hard disk drive
Slow: 15ms

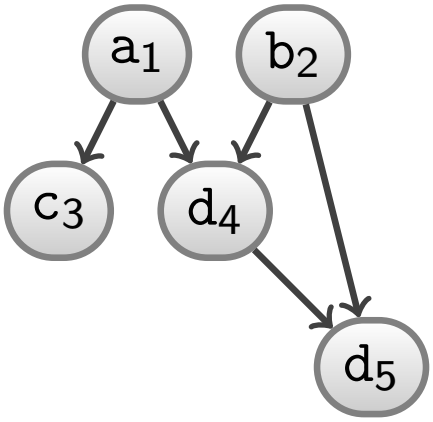
Transfer from the correct position read or write the data
'Fast', small blocks in less than 1μs

Goal: minimize seeks by transferring large blocks

Challenge: graphs



Challenge: graphs



[a, 2, 16, b, 2, 18, c, 0, 20, d, 1, 20, d, 0, 20, 3, 4, 4, 5, 5]

Algorithm

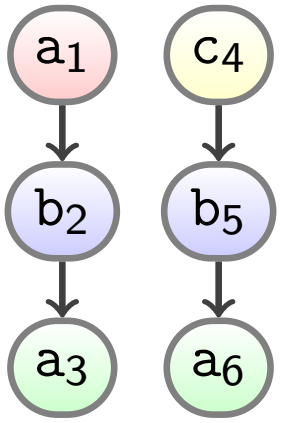
Input: Directed acyclic graph $G = \langle N, E, l \rangle$.
Output: Bisimulation partition of N.

1: P is a partition decision structure
2: **for all** $n \in N$, in reverse-topological order **do**
3: $\quad k \leftarrow \text{Key}(n)$
4: $\quad p \leftarrow \text{Query}(P, k)$
5: $\quad$ **print** (p, n)

Bisimulation key

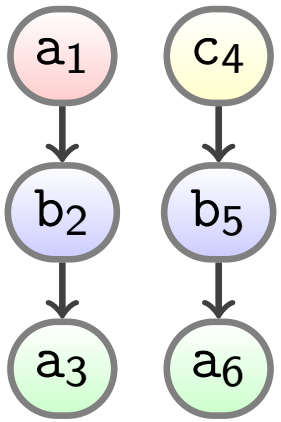
- Nodes with the same key are bisimular equivalent
- Key is a pair (l, S) with
  - l the label of the node
  - S the set of keys of child nodes
- Key is large: replace keys in S by identifiers

Optimize input



Random input a_3; b_2; a_6; a_1; b_5; c_4

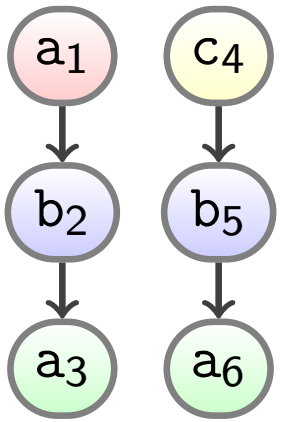
Optimize input



Random input a_3; b_2; a_6; a_1; b_5; c_4

Group on label a_3; a_1; a_6; b_2; b_5; c_4

Optimize input



Random input a_3; b_2; a_6; a_1; b_5; c_4

Group on label a_3; a_1; a_6; b_2; b_5; c_4

Grouping on rank a_3; a_6; b_2; b_5; c_4; a_1

Structural summary

Group nodes on key?

- Therefore we need large partition decision structure P
- Approximate P with hash function

Bisimulation partitioning

- General purpose algorithm
- Reverse-topological ordered directed acyclic graphics
- Expected complexity $O(\text{Sort}(|N|) + \text{Sort}(|E|) + \text{PQ}(|E|))$

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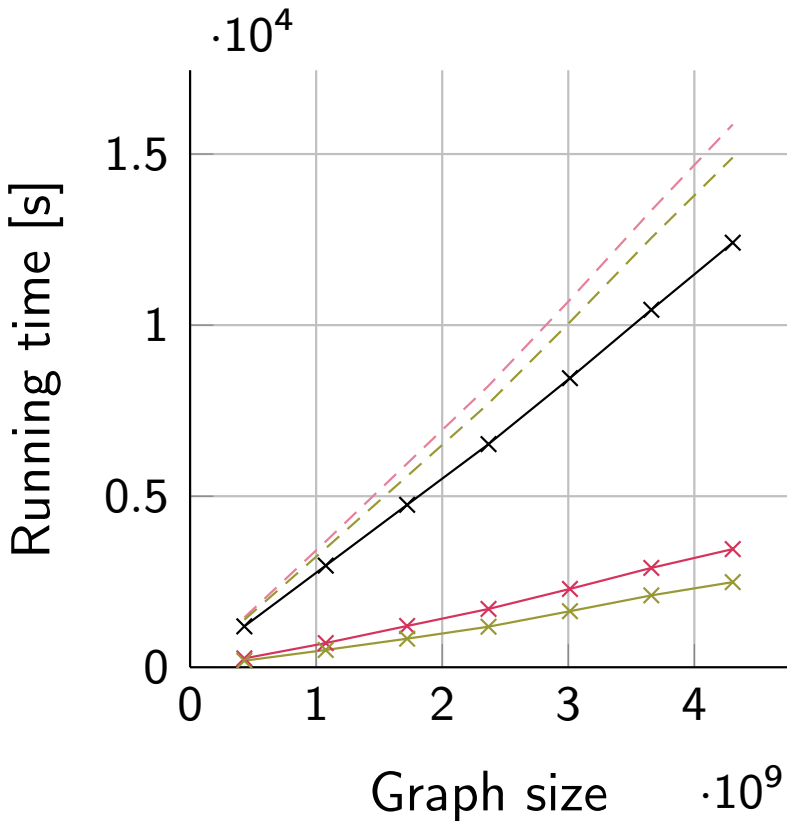
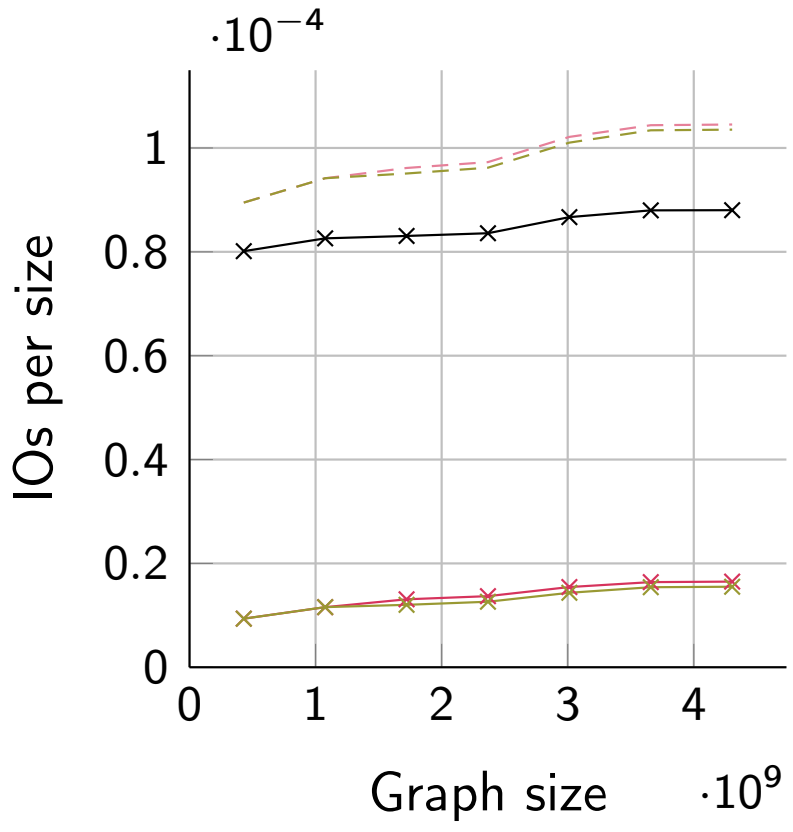
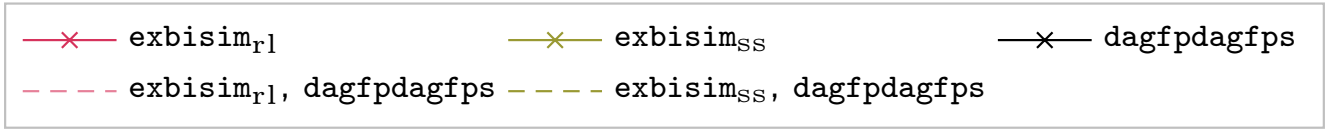
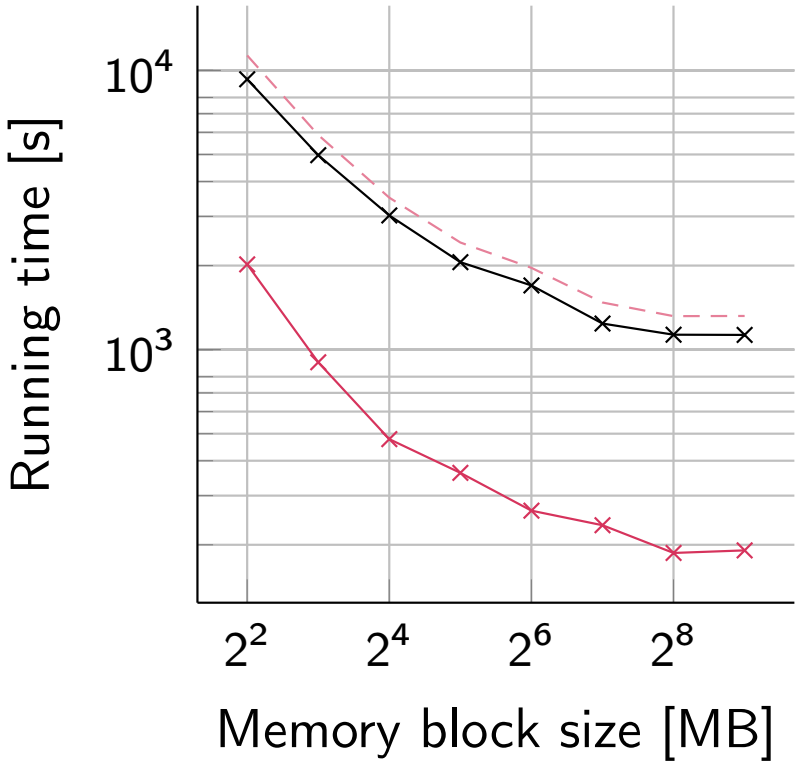
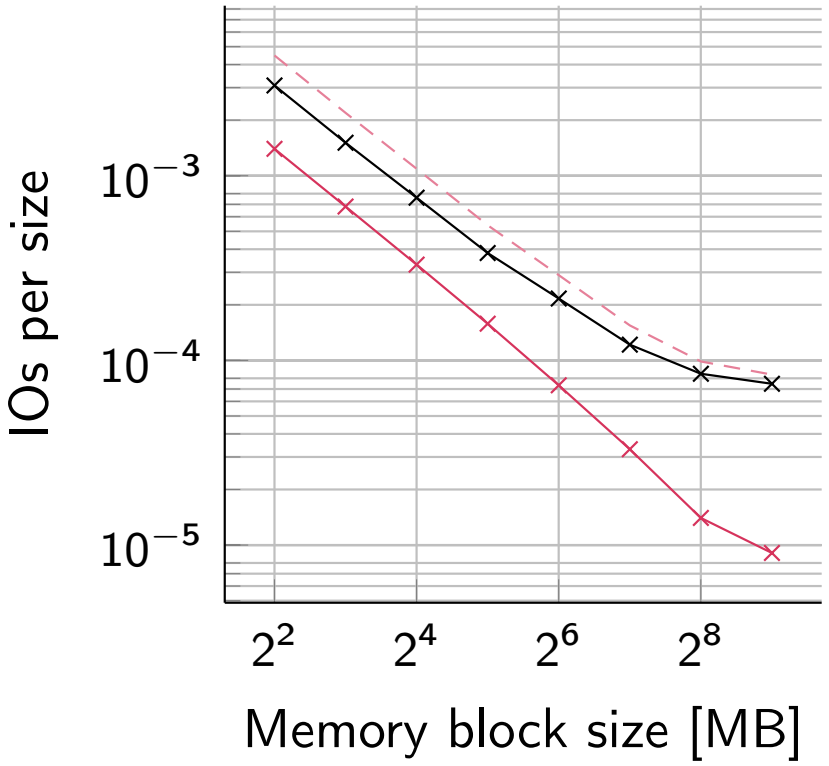
  - Worst case complexity $O(\cdots + \text{Scan}(|N||E|))$


·10^4
1.5
1
0.5
0
Running time [s]
0
1
2
3
4
Graph size
·10^9


·10^−4
1
0.8
0.6
0.4
0.2
0
IOs per size
0
1
2
3
4
Graph size
·10^9


exbisim_rl
exbisim_ss
dagfpdagfps
exbisim_rl, dagfpdagfps
exbisim_ss, dagfpdagfps

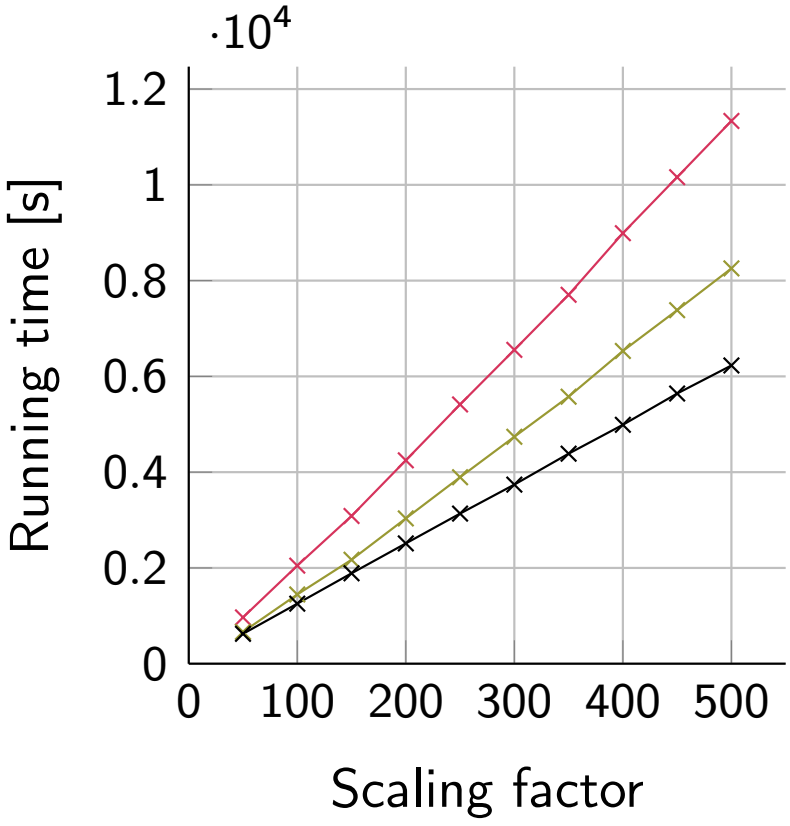

Running time [s]
10^4
10^3
2^2
2^4
2^6
2^8
Memory block size [MB]

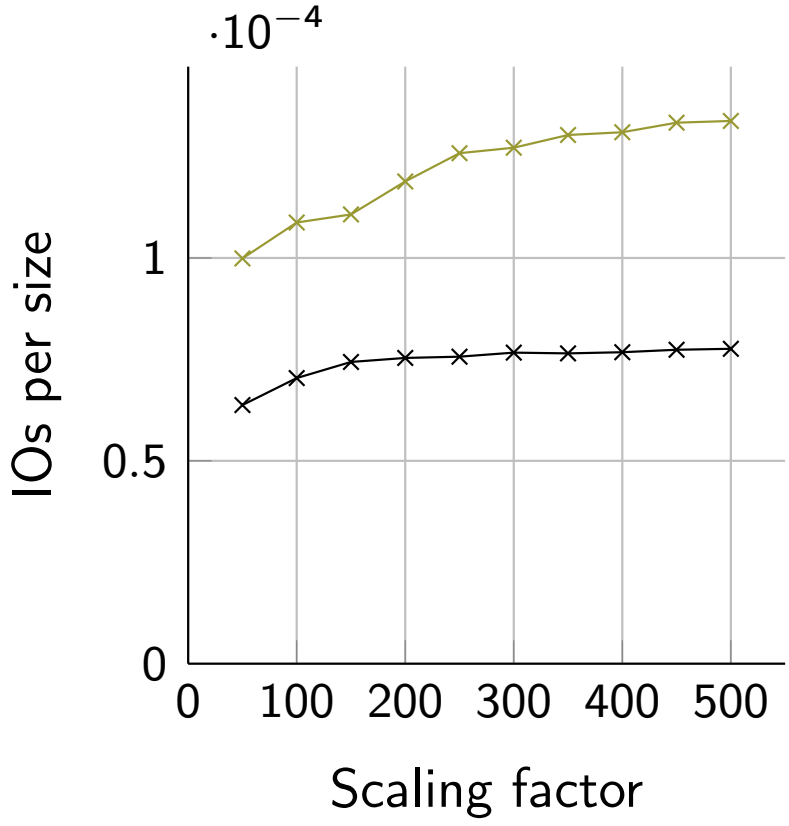

IOs per size
10^{-3}
10^{-4}
10^{-5}
2^2
2^4
2^6
2^8
Memory block size [MB]


exbisim
dagfpdagfps
exbisim, dagfpdagfps

XML index construction

- 1-Index in $O(\text{Sort}(|N|) + \text{PQ}(|N|))$
- A(k)-Index in $O(\text{Sort}(k|N|))$
- F&B-Index by using general algorithm


·10^4
1.2
1
0.8
0.6
0.4
0.2
0
Running time [s]
0
100
200
300
400
500
Scaling factor


·10^−4
1
0.5
0
IOs per size
0
100
200
300
400
500
Scaling factor


exbisim, dagfpdagfps, xmldagfpr
exbisim, dagfpdagfps
xmlbbisim

Why maintenance?

We perform small update to graph; we want an updates partition

- Bisimulation partitioning the entire graph is expensive
- Try to update existing partition

Lower bounds

- Subgraph updates in $O(|N_s| + |E_s|)$

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  - Only possible under 'unreasonable' assumptions

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- Subgraph updates in $O(|N_s| + |E_s|)$
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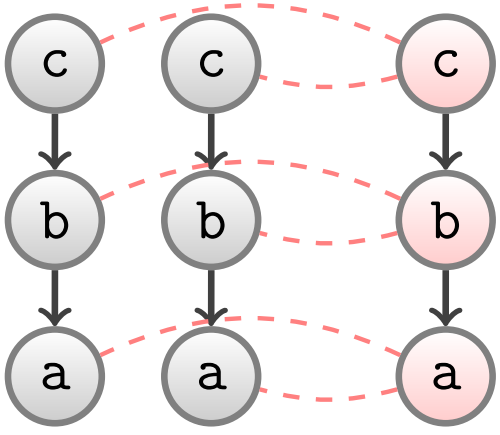
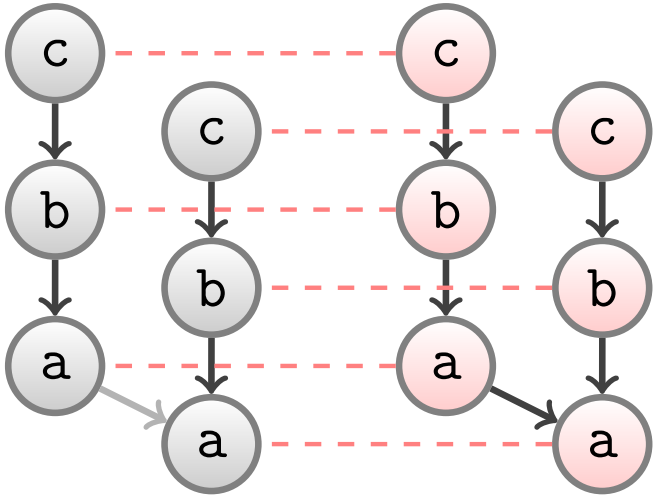
  - Only possible under 'unreasonable' assumptions
- Edge updates in $O(|N| + |E|)$

Edge updates





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- Future work?